

felt to represent the precision of the results for the corresponding measurements on each of copper, silver, and gold.

The comparison of the presently determined  $d \ln c/dP$  for copper with the results obtained by Lazarus<sup>6</sup> is shown in Table IV. In addition, the value obtained by Birch<sup>7</sup> for the pressure variation of the shear modulus of polycrystalline copper,  $d \ln G/dP$ , is listed. It will be noted that the Birch value, representing the derivative of the average shear constant, lies about midway between the  $C$  and  $C'$  values, but that it lies higher than both of the values as determined by Lazarus.

Bridgman's compressibility data are usually expressed in terms of the coefficients  $a$  and  $b$  in the equation

$$\Delta V/V_0 = -aP + bP^2. \quad (7)$$

The quantity  $a$  is related to the isothermal bulk modulus by the equation  $a = (B_T)^{-1}$  and  $b$  is related to the pressure derivative of the bulk modulus by the equation

$$b = \frac{1}{2B_T^2} \left( \frac{dB_T}{dP} + 1 \right). \quad (8)$$

TABLE IV. Comparison of the pressure derivatives of the elastic shear constants of copper with previous data. Units are  $\text{cm}^3 \text{ dyne}^{-1}$ .

Investigator	$d \ln C/dP$	$d \ln C'/dP$	$d \ln G/dP$
Present	3.13	2.48	...
Lazarus <sup>6</sup>	1.13	2.45	...
Birch <sup>7</sup>	...	...	2.76

See reference 6.  
See reference 5.

the values of  $B_T$  given in Table II, and our values of  $dB_T/dP$  (adiabatic), values of  $b$  have been computed. The use of  $dB_s/dP$  instead of  $dB_T/dP$  is not serious; the calculation of the difference from Eq. (6) with the help of standard thermodynamic relations shows that it amounts to about 2%.

Table V compares our values of  $b$  with the Bridgman values (as modified by Slater<sup>20</sup> for copper and silver). The present ones are larger than the Bridgman value in the case of copper, essentially the same for silver, but smaller in the case of gold. The reason for the differences, which are beyond the apparent uncertainty in our work, is not understood. It may be noted that in our acoustic method the quantity under discussion comes from the slope of a raw data plot such as Fig. 1 while in Bridgman's method it comes essentially from the curvature. The fact that the present result is obtained by combining such observations for three waves is admittedly a weakness of the acoustic method but it is not felt to be responsible for the discrepancies.

The pressure derivatives of the elastic constants of copper, silver, and gold are repeated in Table VI, in the form to be used later in the interpretation of the

TABLE V. Comparison of present values of the pressure derivative of the bulk moduli with the Bridgman values. The values are expressed as the constant  $b$  in the equation,  $\Delta V/V_0 = -aP + bP^2$ . Units of  $b$  are  $10^{-12} \text{ cm}^3 \text{ kg}^{-2}$ .

Material	Present $b$	Bridgman $b$
Cu	1.8	1.3
Ag	3.3	3.1
Au	1.3	1.8

results. That is, the pressure derivatives are expressed as  $\Omega dC/d \ln r$ , where the variable  $r$  may be thought of as the distance between nearest neighbor atoms of the crystal and  $\Omega$  is the atomic volume. The relation between the derivative of the elastic constant  $c$  with respect to  $\ln r$  and its pressure derivative is given by

$$dC/d \ln r = -3B_T(dC/dP), \quad (9)$$

and similarly for  $C'$  and  $B$ . We shall hereafter refer to the quantity  $\Omega dC/d \ln r$  as the hydrostatic strain derivative of the corresponding elastic constant. The values of  $\Omega$  used are: Cu 11.81, Ag 17.05, Au 16.96, in units of  $10^{-24} \text{ cm}^3 \text{ atom}^{-1}$ . Table VI illustrates the monotonic variation from copper to silver to gold of all these derivatives, a result to be expected of a homologous series of metals. It is felt that this good intercomparison of the three metals is additional justification of the present results in view of the discrepancies with previous workers shown in Tables IV and V.

#### INTERPRETATION

The elastic constants of a crystal can be expressed as the second derivative of the crystal binding energy with respect to the appropriate strain. The conventional model<sup>21</sup> on which elastic constant calculations are based, considers that the only important contributions to the elastic constants arise from (1) a long-range Coulomb energy, contributing to the shear constants (2) the Fermi energy, assumed in monovalent metals to be a function of volume only and consequently contributing only to the bulk modulus, and (3) a short-range repulsive interaction between neighboring closed shell ion cores. On the usual model, the short-range repulsions are considered to depend only on  $|r|$ , that is, they are assumed to act along lines joining nearest-neighbor atoms. In this section we shall analyze the experimental data from the point of view of this con-

TABLE VI. Hydrostatic strain derivatives,  $\Omega dC/d \ln r$ , of the elastic constants  $B_s$ ,  $C$ , and  $C'$  of copper, silver, and gold. Units are  $10^{-12} \text{ erg atom}^{-1}$ .

	Cu	Ag	Au
$B_s$	-264	-321	-543
$C$	-111	-120	-151
$C'$	-27.4	-33.2	-37.0

<sup>21</sup> N. F. Mott, in *Progress in Metal Physics*, edited by Bruce Chalmers (Interscience Publishers Inc., New York, 1952), Vol. 3, pp. 90-94.

<sup>20</sup> J. C. Slater, *Phys. Rev.* 57, 744 (1940).